International Journal of Theoretical Physics, Vol. 41, No. 8, August 2002 (© 2002)

Frequently Asked Questions About Decoherence

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Received June 23, 2001

We give a short, critical review of the issue of decoherence. We establish the most general framework in which decoherence can be discussed, how it can be quantified, and how it can be measured. We focus on environment induced decoherence and its degree of usefulness for the interpretation of quantum theory. We finally discuss the emergence of a classical world. An overall emphasis is given in pointing at common fallacies and misconceptions.

KEY WORDS: decoherence; quantum mechanics; cosmology.

This paper gives a review of the phenomenon of decoherence. Its emphasis is rather distinct than the one commonly encountered in the literature. Usually the discussion of decoherence is accompanied by an explicit or implicit acceptance of a realist interpretational stance (usually a variation of the Everett stance). However, decoherence as a physical phenomenon is independent of the choice of interpretation and makes sense even in an operationalist perspective, like the Copenhagen interpretation.

This review then takes a minimalist perspective: it focuses on issues that do not require any more specific commitment than that standard quantum theory is a mathematical model that adequately describes experimental outcomes. This is something that all interpretational schemes accept either as a starting point or as a consequence. We therefore refrain from entering into detailed discussion of topics and results that are heavily interpretation-dependent.

In addition, we try to avoid extensive discussion on interpretational issues, but at certain points we have to touch upon such issues, mainly when we find caution about strong claims to be necessary.

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1. WHAT IS MEANT BY DECOHERENCE?

In full generality, decoherence can be defined as the phenomenon by which quantum mechanical systems behave as though they are described by *classical probability theory*. In other words, a quantum mechanical system exhibits decoherence when all typical features of quantum mechanical probability are suppressed. Since probability refers to the statistical properties of systems under study, decoherence refers to behavior that can be inferred from the statistical analysis in a collection of identically prepared systems.[3]

Quantum theory is a theory of complex amplitudes, a fact that is responsible for the distinctive features of quantum probability. In particular

- It implies the existence of off-diagonal elements of the density matrix. Given a density matrix $\hat{\rho}$, the state of a system is specified not only by the probability distribution with respect to a basis $|i\rangle$, $p_i = \langle i|\hat{\rho}|i\rangle$, but also by the off-diagonal elements $\langle i|\hat{\rho}|j\rangle$. The latter have no analogue in classical probability.
- Interference phases appear when we study the probabilistic aspects of quantum systems at successive instants of time. This behavior is highlighted by the two-slit experiment. Consider a system prepared at a state $|\psi\rangle$ and experimental set-up, by which there are two possible alternatives (two slits) at time t_1, represented by the projection operators P_1 and P_2, and a number of possible alternatives represented by Q_i at time $t_2 > t_1$ (the screen). The probability that the system will pass from the slit i at time t_1 and will register at j at time t_2 is equal to

$$p(i, t_1; j, t_2) = \langle\psi|\hat{P}_i \hat{Q}_j \hat{P}_i|\psi\rangle \tag{1}$$

  (We have assumed that the Hamiltonian is equal to zero.) If we consider the probabilities $p(j, t_2)$ that the system is detected in the position j at time t_2 then we see that these probabilities do *not* satisfy the additivity condition

$$p(j, t_2) = p(1, t_1; j, t_2) + p(2, t_1; j, t_2) \tag{2}$$

  The failure of the additivity condition to hold is equal to $Re\langle\psi|\hat{P}_1 \hat{Q}_j \hat{P}_2|\psi\rangle$. This is essentially the interference phase between the two "histories."
- General theorems (due to Bell, 1964, and Wigner, 1976) establish that there exists no probability density that can reproduce all predictions of quantum theory, in particular once that refer simultaneously to noncommuting observables. But even when we restrict ourselves to commutative observables, there exists not a probability theory that can reproduce predictions of multitime probabilities.

[3] Here, we shall not consider decoherence in theories that employ fundamental modifications of quantum theory, such as adding stochastic terms in Schrödinger's equation (Ghirardi *et al.*, 1986; Pearle, 1976). Our focus is on classical behavior and its emergence from the standard quantum mechanical formalism.

Hence, decoherence is mathematically identified through either the diagonalization of a density matrix, or the suppression of interference phases or the ability to adequately model a quantum system by a classical stochastic process.[4]

One should nonetheless distinguish between two very different uses of the word decoherence. One refers to the classicalization of the statistics of a quantum system as a process that takes place *in time* and the other to the emergence of classical behavior inherent in a sufficiently coarse-grained description of a system.

The former notion of decoherence is the one that is more often found in the literature. It is mostly identified with *environment induced decoherence* (Giulini *et al.*, 1996; Joos and Zen, 1985; Paz and Zurek, 2000; Zurek, 1981). Typically this refers to the following situation. A system is prepared in a state that is a superposition of two vectors on the Hilbert space that are macroscopically distinct: $|\psi\rangle = |1\rangle + |2\rangle$. When we let the system evolve, the presence of an environment implies that the state evolves nonunitarily. Hence, the initial pure state evolves into a mixed one. For certain types of environment, it might be the case that even if the coupling of the system to the environment is weak, the density matrix of the system evolves into a state that is a *mixture* of the macroscopically distinct states $|1\rangle$ and $|2\rangle$. In other words, the density matrix of the system becomes rapidly (approximately) diagonal in a given basis. For reasons that will be explained later, the basis in which the density matrix is diagonalized is called the *pointer basis* and the timescale after which this diagonalization has occured is known as the *decoherence time*.

The other type of decoherence refers to the situation where a coarse-grained description of the system can be given in terms of classical probability theory. This is a more general idea of decoherence and as such it refers to intrinsic properties of a physical system (Gell-Mann and Hartle, 1993; Omnès, 1994). What we mean is the following. In general, one cannot access with perfect accuracy the properties of a physical system. One therefore takes recourse to a *coarse-grained* description at

[4] It is important to remark on a common error that arises due to the double semantics of the word coherence. Originally coherence referred to behavior of waves, meaning essentially the absence of spatial dispersion. Because of the wave nature of Schrödinger's equation the word was transferred there. However, the wave function in quantum theory is not a real wave; it is rather a probabilistic object. Hence, quantum coherence is fundamentally defined as a statistical concept, rather than a wave one. In addition it always needs to refer to a particular basis. In field theories, however, observables are of a wave nature themselves. Hence, both notions of coherence can be employed. There is often confusion because of this and absence of wave coherence is often confused with quantum decoherence. This might lead to absurd expressions, such as *propagation of decoherence or local decoherence* in quantum field theoretic or many-body systems. Take, for instance, the coherent states $|z\rangle$ of the electromagnetic (EM) field. One can choose for $z(\mathbf{x})$ functions that correspond to classical field configurations *that exhibit spatial coherence*. (The use of the word "coherent" for the name of coherent states refers to classical coherence of the EM field). Let us take two of them $z_1(\mathbf{x})$ and $z_2(\mathbf{x})$. Each state exhibits classical coherence but trivial quantum coherence (with respect to the phase space basis). The state $|z_1\rangle + |z_2\rangle$ is highly coherent quantum mechanically, but incoherent classically. The state $|z_1 + z_2\rangle$ exhibits trivial quantum coherence and no classical coherence. Finally, the state $\frac{1}{2}(|z_1\rangle\langle z_1| + |z_2\rangle\langle z_2|)$ exhibits neither classical nor quantum coherence.

a level that is accessible to us: mathematically, this means that the properties of the system are described by projection operators P to subspaces of the Hilbert space that are not one-dimensional (TrP quantifies the degree of coarse-graining). As a result of the description in terms of coarse-grained observables, the effect of the interference phases might be suppressed. In this case, the system can be described by a stochastic process (Anastopoulos, 2001; Gell-Mann and Hartle, 1993; Hartle, 1993).

The environment induced decoherence is a special case of this more general characterization of decoherence. If the individual quantum system is described by a Hilbert space H_S and the environment by a Hilbert space H_E, the combined system is described by a Hilbert space $H_S \otimes H_E$. The coarse-graining consists in the consideration of operators only of the type $P \otimes 1$, i.e. ones that project only to the system's Hilbert space.

We shall, henceforth, refer to the enironment induced decoherence as extrinsic (since it is caused by an external agent) and the second type as intrinsic (since it appears as a consequence of the basic properties of the system).

2. HOW IS DECOHERENCE QUANTIFIED?

A naive estimation of the degree of decoherence (with respect to a basis) comes from the comparison of the diagonal to the off-diagonal elements of the density matrix of the system. Hence a criterion is that $|\hat{\rho}_{ij}/\hat{\rho}_{ii}| \ll 1$ for all i, j. Whenever this is true, the diagonal elements $\hat{\rho}_{ii}$ define a probability distribution $p(i)$ and the basis $|i\rangle$ is a pointer basis for this system.

This, however, is very imprecise. First, it is classically reasonable that for some i, $\hat{\rho}_{ii} = 0$. In this case, a simple comparison with the off-diagonal elements would not be sufficient. A more sharp criterion can be phrased in terms of information theory (Anastopoulos, 1999). Given a probability distribution $p(i)$, we can define the corresponding Shannon information as

$$I[p] = -\sum_i p(i) \log p(i) \tag{3}$$

In order to discuss classicality we need to compare this with a quantum mechanical information quantity: the von Neumann entropy

$$S[\hat{\rho}] = -\mathrm{Tr}\, \hat{\rho} \log \hat{\rho} \tag{4}$$

Now it is easy to verify that if we set $p(i) = \hat{\rho}_{ii}$, the following inequality holds

$$I[p] - S[\hat{\rho}] \geq 0 \tag{5}$$

with equality only if $\hat{\rho}$ *is diagonal in the basis* $|i\rangle$.

We can, therefore, consider as a criterion for approximate diagonalization the condition $I[p] - S[\hat{\rho}] \ll 1$. In order to establish the decoherence due to environment one has to verify that $I - S$ rapidly falls close to zero.

However, such criteria, which refer to a given basis, are not always practical or even physically meaningful. First, one does not know a priori the basis upon which the diagonalization will take place, if at all. Even if this is the case there is no guarantee that the decoherent behavior of the system will be present in all physically realized measurements. For instance, a density matrix that is approximately diagonalized in position, can give highly nonclassical results for measurements of momentum.

One issue that is forgotten in many analyses is that of the *robustness* of the pointer basis. By this we mean that the behavior of the system should be "classical" not only with respect to the operators that correspond to the basis, but to a larger class of them. An elementary example is that of a free particle ($H = p^2/2m$). In the long time-limit the state becomes approximately diagonal in the momentum basis, but this is no indication of "classical behavior." If one starts with a superposition of two states, each of them localized in position but with large separation of their centers, measurements of almost all observables but position would exhibit strong interference, even though the state approaches (weakly) a delta function in momentum. How this is problematic can also be understood in light of the following remarks.

There are many ways in which the term "basis" is used. Our analysis refers to discrete bases, i.e. proper orthonormal bases in the Hilbert space. But in studies of decoherence it is often taken to imply continuous bases like momentum. In this case our criteria for decoherence need to be substantially modified. (For instance, the corresponding entropies (4) are not bounded from below.) One might construct a discrete orthonormal basis by coarse graining a continuous one—as, for instance, von Neumann employed Gaussians to construct approximate position observables with discrete spectrum (Neumann, 1996). But the procedure is far from unique and the degree of decoherence depends on the choice of coarse graining. One would then have to establish a relative insensitivity to such a choice in order to unambiguously identify decoherence. For this reason, approximate diagonalization in a given continuous basis is not by itself adequate to infer that the system is effectively decoherent and additional criteria have to be established. One such idea is the one of the "predictability sieve" (Paz and Zurek, 1999; Zurek *et al.*, 1993), i.e. to look for bases consisting of states that are minimally entangled with the environment in the course of evolution. But generically, such states form over complete bases on the system's Hilbert space and cannot provide by themselves an unambiguous basis. It is more precise to talk of a "halo" of nearby pointer bases (Anglin and Zurek, 1996) rather than a precise one and to demand approximate diagonalization in all bases of the halo.

It is, therefore, necessary to employ criteria that refer to probabilistic aspects of the system that are *not sensitive to an arbitrary choice of basis*. These are provided by the phase space description of the system, which is inherent in the structure of the canonical commutation relations. Since phase space observables exhaust the physical content of a quantum system, they can provide a most robust criterion for decoherence. In particular, the most useful tool in this regard is the Wigner function. This is defined as a pseudoprobability distribution on the classical phase space. It is defined in terms of the density matrix $\hat{\rho}$ as

$$W(q, p) = \mathrm{Tr}\,(\hat{\rho}\hat{\Delta}(q, p)) \tag{6}$$

where $\Delta(q, p)$ is an operator defined by

$$\hat{\Delta}(q, p) = \int \frac{du\,dv}{2\pi} e^{-iqu - ipv}\, e^{iu\hat{Q} + iv\hat{P}} \tag{7}$$

The Wigner function is not positive, and hence not a true probability distribution. It can also take negative values. Quantum coherence manifests itself in oscillations around zero at the scale of $\hbar$. The only pure states that give rise to positive Wigner functions are the Gaussian ones. For instance a quantum state that is a superposition of two Gaussians with different centers, like

$$\psi(x) = \frac{1}{\sqrt{2}\pi^{1/4}\sigma}\left[e^{-x^2/2\sigma^2} + e^{-(x-L)^2/2\sigma^2}\right] \tag{8}$$

The corresponding Wigner function is

$$W(q, p) = \frac{1}{\sqrt{2}\sigma} e^{-p^2/\sigma^{-2}} \times \left(e^{x^2/\sigma^2} + e^{-(x-L)^2/\sigma^2} + 2\, e^{L^2/4\sigma^2} e^{-x^2/2\sigma^2 - (x-L)^2/2\sigma^2} \cos Lp\right) \tag{9}$$

The first two terms in (9) correspond to a mixture of two classical probability distributions centered around $x = 0$ and $x = L$. The third term, however, exhibits strong oscillations and has a prefactor that increases exponentially with the degree of separation between the two Gaussians. When $L/\sigma > 1$ the Wigner function of the oscillating term causes the Wigner function to take negative values. It is then natural to consider the suppression of such oscillating terms as a sign of decoherence. This is equivalent to the suppression of the off-diagonal terms in a phase space basis (e.g. coherent states) and corresponds to the statement that the quantum system can be described by a probability distribution (a positive definite Wigner function).

The study of the Wigner function is a good measure for the case of environment induced decoherence. For the case of intrinsic decoherence, the best prescription comes from the consistent (decoherent) histories approach to quantum theory (Gell-Mann and Hartle, 1990, 1993; Griffiths, 1984; Hartle, 1993; Omnès, 1988, 1994).

A history α is defined as a sequence of properties of the system at successive moments of time. Hence it is represented by a sequence of projection operators $\hat{P}_{t_1}, \ldots, \hat{P}_{t_n}$. The information about interference and probabilities is encoded in the *decoherence functional*, a complex valued function of pairs of histories. This is given by

$$d(\alpha, \beta) = \mathrm{Tr}\,(\hat{C}_\alpha^\dagger \hat{\rho}_0 \hat{C}_\beta) \tag{10}$$

where

$$\hat{C}_\alpha = e^{i\hat{H}t_1}\,\hat{P}_{t_1}\,e^{-i\hat{H}t_1}\cdots e^{i\hat{H}(t_n-t_{n-1})}\,\hat{P}_{t_n}\,e^{-i\hat{H}(t_n-t_{n-1})} \tag{11}$$

and $\hat{\rho}_0$ is the initial state of the system. The analysis of the two-slit experiment, we gave earlier, suggests the following natural consideration. If in an exhaustive and exclusive set of histories, we have the property

$$d(\alpha, \beta) = 0 \quad \alpha \neq \beta \tag{12}$$

then there exists a probability measure for this set of histories, given by $p(\alpha) = d(\alpha, \alpha)$. This means that these histories can be described by probability theory. Typically, decoherent sets contain coarse-grained histories.

Hence, the construction of the decoherence functional can provide a good criterion for decoherence. However, the objection of robustness can be raised in this case as in the single time description. Consistency of an arbitrary set of histories might have little to do with classicality of a large class of observables. For this reason it is perhaps best to consider the decoherence functional on phase space. This can be obtained by the Wigner transform. Such a construction is given in Anastopoulos (2001), employing the techniques of continuous-time histories (Isham, 1994; Isham and Linden, 1995; Isham *et al.*, 1998; Savvidou, 1999a,b): it provides a natural way to determine decoherence of the most general type. Alternatively one can employ information-theoretic quantities (Halliwell, 1993a,b).

3. HOW CAN DECOHERENCE BE MEASURED?

Decoherence is a probabilistic concept. Therefore, it is only in a *statistical* sense that we can talk about its presence in a physical system. In other words, we cannot make any conclusions about decoherence in the study of an *individual* quantum system, since the concept does not make any operational sense there.

The only operational way of identifying decoherence lies in the consideration of the statistical behavior in a *collection of identically prepared systems*. This means that we need to reconstruct from measurements (in different individual systems) the statistical properties of the state in which the ensemble is prepared, study its evolution in time, and establish the presence of decoherence by using some quantitative criterion. Let us explain this in some detail. First, it is *not possible* to talk about decoherence by focusing on the evolution of a *single* variable, say $\hat{A}$. The reason is the following: if we write its spectral decomposition then

$$\hat{A} = \sum_i \lambda_i |i\rangle\langle i| \tag{13}$$

The expectation value will read $\langle \hat{A} \rangle = \sum_i \lambda_i \rho_{ii}$ in terms of the density matrix ρ of the system. Clearly, measurements of $\hat{A}$ allow us to probe *only diagonal elements of the density matrix*. If we are to ascertain decoherent behavior this is not sufficient as we would need to make statements about the off-diagonal elements as well.

Hence we conclude that in order to claim that a system classicalizes we need to have access to the values of noncommuting observables at the same time. This is not possible for a single system, but there is no problem when we consider ensembles of identically prepared systems. This means that we prepare a collection of systems in the state ρ and then at a moment of time we can perform measurements of different noncommuting observables at different individual systems. This way we can reconstruct the statistical properties of the system in the prepared state through standard procedures of data analysis and state estimation (see, e.g., Helstrom, 1976; Holevo, 1982). Performing such a series of measurements at different moments of time, we might notice the suppression of off-diagonal terms in some basis, which will be a sign of decoherence. It is the author's opinion that there is no failproof way to establish whether a particular physical system decoheres or not, except for the state estimation based on measurements of incompatible observables at successive moments of time. In many-body systems, in particular, one often confuses the wave notion of coherence with the quantum one.

This is definitely true for decoherence induced by the environment. There is a situation, however, that one can establish decoherence of individual systems: this is the case of (approximate) determinism. In this case, the internal dynamics of the system are such that the coarse-grained quantum mechanical observables are correlated according to a *deterministic equation of motion*. This is believed to arise for a large variety of quantum systems: such would be the case of the emergence of classical mechanical laws from the underlying quantum theory. In that case, the existence of almost complete predictability ensures effective classicality in the coarse-grained description of the quantum system.

4. THE "PHENOMENOLOGY" OF ENVIRONMENT INDUCED DECOHERENCE

By phenomenology we mean the theoretical study of certain open quantum systems that are thought to provide a guide for the behavior of quantum systems decohering under the action of an external environment.

The first studies emphasized the rapidity of the decoherence process for macroscopic systems in an incoherent (thermal) environment. A simple model by Joos and Zeh (1985) established a much quoted result: that even if the environment is of so low temperature as the cosmic microwave background, a superposition of two states with difference in their centers of the order of 1 cm, for a macroscopic body (mass $m \sim 1$ kg), would lose its coherence in a timescale of the order of

10^{-23} s. In this sense, environment induced decoherence of macroscopically distinct superpositions is said to be among the fastest processes in nature.[5]

The main paradigm, though, for studies of this type is the quantum Brownian motion models (Unruh and Zurek, 1989). They consist of a particle with mass M and moving in a potential $V(x)$ (the system), in contact with a large number of harmonic oscillators (the bath) (Caldeira and Leggett, 1983; Grabert *et al.*, 1988; Hu *et al.*, 1992). The Hamiltonian of the system is therefore

$$H = \frac{p^2}{2M} + V(x) + \sum_\alpha \left(\frac{p_\alpha^2}{2m_\alpha} + \frac{1}{2} m_\alpha \omega_\alpha^2 q_\alpha^2 \right) + \sum_\alpha c_\alpha q_\alpha x \tag{14}$$

We trace out the contribution of the environment and study the evolution of the reduced density matrix. The contribution of the environment is contained in the *spectral density*, a function $I(\omega)$ defined by

$$I(\omega) = \sum_\alpha \frac{c_\alpha^2}{2m_\alpha \omega^2} \delta(\omega - \omega_\alpha) \tag{15}$$

There are three important parameters that characterize physical spectral functions: a high energy cut-off Λ, an exponent characterizing the low-frequency behavior and an overall multiplicative constant γ that incorporates the effects of dissipation. Typically, one writes the spectral density as

$$I(\omega) = \gamma \omega^s \, e^{-\omega^2/\Lambda^2} \tag{16}$$

When the initial state of the total system is assumed factorized, the density matrix of the environment is Gaussian, and $V(x)$ is quadratic (or zero), one can exactly solve for the propagator of the reduced density matrix of the system and construct the master equation that describes its evolution.

A well studied case is the Fokker–Planck limit: the environment is *ohmic* ($s = 1$) and in a thermal state with temperature $T \gg \Lambda$. In this case the reduced density matrix satisfies a *Markov* differential equation known as the Kramers equation,

$$i \frac{\partial}{\partial t} \rho = \left[\frac{p^2}{2M} + V(x), \rho \right] - \gamma [x, \{p, \rho\}] - 2iM\gamma T [x, [x, \rho]] \tag{17}$$

In this regime, the evolution of a state of the form (8) shows an exponential suppression of the oscillating term in the corresponding Wigner function with the decay rate of the form of $e^{-2M\gamma T L^2 t}$, and hence a decoherence time $t_{\mathrm{dec}} = (M\gamma T L^2)^{-1}$.

[5] This is valid, of course, if we assume that such superpositions can be created in the first place. This assumption is interpretation-dependent, and is natural in realist interpretations of quantum theory. However, the Copenhagen interpretation or any operational scheme for quantum theory need not accept the possibility of existence of such states, since the macroscopic classical world is assumed, *a priori*.

In this model, there are three important timescales that are generic in many open quantum systems.

There is the inverse cut-off time Λ^{-1}, which describes the immediate response of the reservoir to the quantum system. For times $t < \Lambda^{-1}$ the factorized initial condition often gives a bad approximation, since at these times the evolution is very sensitive on high energy correlations between system and environment (Grabert *et al.*, 1988; Hu *et al.*, 1992), which are operationally uncontrollable. There is the classicalization time $t_{\mathrm{cl}} = (M\gamma T)^{-1/2}$ (Anastopoulos and Halliwell, 1995; Halliwell and Zoupas, 1995). This is an upper limit to decoherence time; in fact one can show that after this time, thermal fluctuations overcome the quantum ones and that the system is adequately described by the evolution of a classical probability distribution. This timescale governs the rate by which quantum phases move from the system to the environment. Finally there exists the relaxation time γ^{-1}, which governs the rate of energy flow from the system to the environment. For realistic values of the parameters, we have that $\Lambda^{-1} \ll t_{\mathrm{cl}} \ll \gamma^{-1}$. This separation of timescales conforms to the most clear-cut case of environment induced decoherence: the timescale that governs a purely quantum process (the escape of phases to the environment) is much smaller than the timescale of the classical energy exchange. Hence, even when one can consider the system as almost closed ($\gamma^{-1} \to \infty$), the loss of quantum phases is still important and sufficient to classicalize the system.

In other regimes, these three timescales are not widely separate. For instance, at low temperature (and ohmic environment) decoherence appears within a timescale of Λ^{-1}. However, in this regime the use of a factorized initial condition is not necessarily physical and one should consider the possibility that the decoherence phenomena predicted are artifacts of an unphysical initial condition, or at least that decoherence is contingent on the initial correlations of the system to the environment (Hu *et al.*, 1992).

The ohmic case is the standard by which to judge quantum Brownian motion. The cases of subohmic ($s < 1$) and supraohmic ($s > 1$) environments are different. In the former the response of the system to the environment is much stronger and decoherence is more efficient, while the opposite behavior is manifested in the latter case.

There is a sense in which environment induced decoherence is highly dependent on the infrared behavior of the environment. Intuitively the reason is that the information of the quantum phases leave the individual system, be spread in the environment, and not return. In order for this to happen it is not only necessary that the environment be *large*, but its recurrence time should also be large. So, for instance, if the environment consisted of a large number of harmonic oscillators with the same frequency ω, there would typically be a recurrence time of the order of ω^{-1} and the quantum phases would reappear in the quantum system after this time. (In a sense it is similar to the Poincaré recurrence of classical mechanics.)

It is, therefore, essential that the recurrence time be long: in harmonic oscillator baths this is guaranteed by the strong presence of infrared modes $\omega \to 0$ in the spectral density (Kupsch, 2000).

The behavior we analyzed in quantum Brownian motion is often considered as paradigmatic (Omnés, 1997). Indeed it is a good approximation to a large class of environments, even though it is a very special model. In addition to the spectral density, this behavior is dependent on the choice of the initial condition (thermal state) and the coupling between system and environment. There are no studies of initial states, substantially from thermal one, but it seems reasonable that this classicalization behavior is typical for sufficiently "classical" initial conditions as thermal states (or vacuum for $T = 0$) and would not persist in states with quantum behavior such as squeezed states.

Also, in quantum Brownian motion the system couples to the environment in the position basis. A resonant type of coupling (as appears for instance in atom–field interaction (Anastopoulos and Hu, 2000)) leads to decoherence in the energy basis and within a timescale that is of the same order of magnitude as the relaxation time. According to our previous discussion, this is reasonable since a resonant type of coupling effectively selects a part of the environment's modes as relevant (the ones around the resonance's frequency) and misses the important contribution of the infrared sector. It is a matter of convention whether one will call this type of behavior as lying within the domain of environment induced decoherence, because decoherence is in a sense trivial. The flow of quantum phases to the environment is not distinct from the energy flow. As such, the two phenomena cannot be considered as separate.

More general (nonlinear) systems exhibit more complicated behavior (Anglin *et al.*, 1996; Paz and Zurek, 2000): this is a consequence of many time- and lengthscales that characterize them. One interesting possible consequence is that decoherence is saturated at a distance of separation (in the position basis). Spin baths are thought to be agents of decoherence more effective than bosonic ones at low temperature (see Stamp and Prokov'ef 2000, and references therein). However, such calculations often involve a perturbative expansion, which sometimes is not a good indicator of decoherence behavior.

In light of these remarks, we can say that in order for a environment induced decoherence to be manifested in a system interacting with an environment the following requirements must be met:

i. The environment itself has to be in a "classical" state, like a thermal state, or vacuum. Here classical refers to its behavior with respect to its Wigner function description.
ii. The system–environment coupling should be in a continuous (position or momentum) basis, rather than a discrete one.

iii. The spectral density of the environment has to grow slowly in the infrared regime. In effect, this means that the environment responds more slowly in the appearance of the quantum system and hence the quantum phases are lost before the steady rate of energy flow commences.

5. WHAT DOES DECOHERENCE IMPLY FOR THE INTERPRETATION OF QUANTUM THEORY?

Decoherence as a phenomenon is, in general, insensitive to the interpretation one decides to employ for quantum theory. The transition from quantum behavior to classical statistics makes sense both in an operational setting (like the Copenhagen interpretation) or a realist one.

However, it has been historically associated with realist interpretations of quantum theory and more particularly with the many-worlds interpretation or its offspring. These attempt to interpret the quantum mechanical formalism as though it refers to individual quantum systems, rather than statistical ensembles. These interpretations suffer from a severe problem: the fact that quantum systems cannot be said to possess a given property without making reference to the way one reasons in order to verify the truth of this assertion. This is a corollary of the noncommutativity of observables in quantum theory, or more precisely of the nondistributivity of the lattice of propositions and is known as the Kochen–Specker's theorem (Kochen and Specker, 1967). If one wants to talk about definite properties of an individual quantum system, one always has to make reference to a Hilbert space basis (or more generally an Abelian subalgebra of observables). And the initial focus of the many-worlds interpretation was to examine the presence of such bases (at least) in measurement situations.

In a realist interpretation both the measured system and the device are described by wave functions, which are elements of the Hilbert spaces H_S and H_M respectively. The Hilbert space of the combined systems is then $H_S \otimes H_M$.

A basis $|R\rangle$ in the H_M is assumed to be associated to the measurement device, each possible R corresponding to a different value of the pointer in the device. Also, let $|i\rangle$ denote a basis in H_S, with corresponding values α_i of the observable $\hat{A}$ for the system. Initially, the total system lies in an uncorrelated state $|\Psi\rangle = |\psi_0\rangle \otimes |R_0\rangle = (\sum_i c_i^0 |i\rangle)|R_0\rangle$, with $|\psi_0\rangle$ the initial state of the system c_i^0 its coefficients in the basis $|i\rangle$, and $|R_0\rangle$ the initial position of the pointer. As a result of the interaction the state at the end of the process will be $|\Psi\rangle = \sum_{Ri} d_{iR} |i\rangle |R\rangle$. Even if there exists perfect correlation between initial eigenstates of the system and final values of the pointer (i.e. if $|i\rangle|R_0\rangle \to |i\rangle|R_i\rangle$, where R_i is uniquely determined by the value i) any superposition in the $|i\rangle$ basis will lead to an entangled state for the total combined system. The apparatus is then not in a definite macroscopic configuration, which then implies that we cannot ascribe a property like the value of the pointer to it. This is the essence of the measurement problem, also known as the *macroobjectification* problem. One solution is the *infamous* wave packet reduction, proposed by von Neumann, which states that after the measurement the state of the total system reduces to a mixture (Neumann, 1996) $\sum_i |d_{iRi}|^2 |i\rangle\langle i| \otimes |R_i\rangle\langle R_i|$, which is interpreted in terms of classical statistics. This process is postulated in an ad hoc manner and its origin is seemingly mysterious.

This is accompanied with another severe problem. How can we make sure that a given apparatus is associated to a particular observable? In other words, why would the wave packet reduction take place in the $|R_i\rangle$ basis, which is perfectly correlated with i and not in any other? There seems to be an arbitrariness in the choice of the basis in which a mixture collapses.

The environment induced decoherence has arisen as a possible solution for the second problem (Zurek, 1981, 1982) and is also often claimed to provide a solution to the more fundamental macroobjectification problem. The essential argument is that any apparatus is in contact with the environment; the environment induces a rapid diagonalization of the density matrix of the apparatus in a fixed basis. Hence, the wave packet reduction always takes place with respect to this basis for the total system. This (together with the interaction Hamiltonian between measured system and apparatus) is then the factor that determines what is the actual correlation between the basis of the measured system and the pointer basis of the apparatus. This proposal is a very natural solution to the problem of the arbitrariness of the pointer basis. However, there are a number of points that need to be addressed before this answer is taken as definite.

First, since the diagonalization due to the environment is not exact, for a given state of the environment there exists a number of possible pointer bases for the apparatus, that are close (with respect to some natural distance in the apparatus's Hilbert space). It has to be shown that the correlation between the measured system and the apparatus is largely insensitive to the precise choice one makes for the pointer basis.

Second, an apparatus can measure the same physical quantity in very different situations, which can correspond to very different states of the environment. It has therefore to be shown that the choice of the pointer basis is robust within reasonable variations of the environment's state and constituents. This has not yet been established from the study of the existing simple models.

The idea of the environment induced decoherence provides a good programme toward explaining the correlation of pointers in apparatuses and properties of measured systems in the realist interpretations of quantum theory, even though it cannot yet be taken as a definite answer. However, it is often claimed that environment induced decoherence provides by itself a solution of the macroobjectification problem (e.g. Zurek, 1991). This statement is not true for the following reasons.

We saw earlier that the root of the macroobjectification problem is that the final state of the combined microscopic system and apparatus corresponds to no definite macroscopic superposition. The presence of the environment would typically make the final state mixed. Let us ignore for the moment an immediate objection: that there is no guarantee that the general state of the environment will allow it to play the role of a decohering agent and that the resulting diagonalization will be robust. There is no reason for the resulting pointer basis of the combined system to be factorizable and hence to correspond to macroscopically distinct properties for the quantum system. This can be explicitly seen in calculations in toy detector models (one explicit calculation is found in Zoupas, 1997). Also, there exists a theorem that establishes that the combined system will not exhibit states that are macroscopically definite under a wide variety of time evolution laws. Such a theorem was first proved by d'Espagnat (1989) and recently strengthened by Bassi and Ghirardi (2000).

One concludes therefore that environment induced decoherence cannot *by itself* explain the appearance of macroscopic definite properties and a realist interpretation of quantum theory still needs an additional postulate to account for macroobjectification, as in von Neumann's measurement theory, the Everett stance, the collapse models, or in consistent histories.

6. HOW IS THE CLASSICAL WORLD EXPLAINED?

One is often tempted to explain the emergence of the classical world as a result of environment induced decoherence, in the sense that the definite properties that can be attributed to objects of our experience emerge as a result of the effect of the environment. However, a sufficiently classical behavior for the environment seems to be necessary if it is to act as a decohering agent and we can ask what has brought the environment into such a state ad-infinitum. One would then be forced to employ quantum theory at increasingly large scales and at the end such a question can only be answered at the level of considering the question at the level of the universe.[6]

The issue is then raised at the level of quantum cosmology. Since the universe is a closed system, the reasonable way to explain classicality in this framework is by identifying some degrees of freedom that are all pervading and sufficiently autonomous. There are many such proposals taking as a fundamental environment: the matter field fluctuations (Halliwell, 1989; Kiefer, 1989; Padmanabhan, 1989), the gravitational field (Anastopoulos, 1996; Kay, 1998), high energy

[6] Such is a realist answer. For the Copenhagen interpretation the classical world is usually taken as something in which the quantum description is not applicable. Strictly logically, the Copenhagen interpretation does not need to explain the emergence of the classical world; but in this case it has to admit the failure of quantum theory to be a universal theory and also, how a macroscopic system of quantum constituents behaves classically.

modes (Lombardo and Mazzitelli, 1996), the higher order correlation functions (Anastopoulos, 1997; Calzetta and Hu, 1993, 1995), etc. The choice is often taken according to the convenience of the question one wants to study.

However, there does not exist a conclusive argument why some particular degree of freedom ought to play the degree of a decohering environment. In the case of the gravitational field its universality and the lack of a theory of quantum gravity make it a plausible candidate. Gravity is often stated as a possible cause of fundamental modifications in quantum theory (Karolyhazy *et al.*. 1988; Penrose, 1988) (not necessarily of the nature of environment induced decoherence as in Anastopoulos, 1996, and Kay, 1998). However, if one insists on the environment satisfying the laws of quantum theory one still is entitled to ask, how come that it is in a state that is able to cause decoherence. The only conceivable answer is the postulation of a special initial condition. In addition, the question is raised, what sense does it make to talk about separation of degrees of freedom in highly nonlinear theories, such as general relativity coupled with matter.

An environment induced decoherence seems therefore not to be able to account convincingly about the emergence of classical behavior. But perhaps classicality arises as a result of intrinsic decoherence. This is indeed a main point of the consistent histories approach; however it does not necessitate adherence to this interpretation. To see how it works let us examine a simple example. Consider a system that is adequately described by Schrödinger's equation in one dimension. Its phase space would be $\mathbf{R}^2$. In this phase space positive operators that correspond to phase space cells can be defined and for sufficiently large areas of the enclosed cell they are close to projection operators. Each of these operators P_C can be said to correspond to the statement that the system is with high accuracy within the phase space cell C. Now, as a result of the quantum dynamics, it might be that for sufficiently large cell C the operators P_C evolve as $P_{C_0} \to P_{C_t}$ with high accuracy. This implies that properties of the system (whenever they are definite) evolve according to approximately deterministic equations of motion. This has been proved to hold for a large class of potentials (Omnès, 1988, 1994). Clearly, here classicality is a result of an approximate determinism of sufficiently coarse-grained quantities, which appear naturally from the formalism.

This was an idealized example for the case of a system with a single degree of freedom. One is, however, interested in explaining the classicality of systems that consist of a large number of degrees of freedom. In this case there are certain types of coarse-grainings that might effect a deterministic description.

First, one might choose to focus on the evolution of *hydrodynamic variables*, i.e. variables such as energy density and particle density. There are some reasons that make plausible that such quantities exhibit classical—in fact almost deterministic—behavior in many-body systems. In particular, the densities $\rho(x)$ of conserved quantities (e.g. energy, charge), when integrated into finite volumes, vary much slower than other currents, since by virtue of the conservation equation $\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$, we have

$$\frac{\partial}{\partial t} \int_V d^3 x \rho(x) = -\int_V d^3 x \nabla \cdot \mathbf{j} = \int_{\partial V} d\sigma \cdot \mathbf{j} \tag{18}$$

and given a sufficiently regular volume V of characteristic length scale l, the density varies with the *area* of its boundary, i.e. with l^2, unlike other densities that vary with l^3. So, coarse-graining in position, for sufficiently large values of l, will tend to make averaged densities of conserved quantities changing more slowly than other averaged densities. Given the fact that conserved quantities decohere trivially (Hartle *et al.*, 1995), this is an indirect suggestion that these variables would be the first to examine for effective classical behavior. There are some elementary models that support this assertion (Brun and Halliwell, 1996; Brun and Hartle, 1999; Calzetta and Hu, 1999; Halliwell, 1998, 1999); however, we still lack a conclusive general argument. In particular, it is not yet clear whether special initial conditions are needed in order to guarantee the decoherence of hydrodynamic variables.

Clearly, should it be shown that hydrodynamic variables habitually decohere, would go a long way toward explaining the origin of a classical world in the cosmological context first, but also for general macroscopic systems (e.g. rivers, planets, rocks, . . .). In particular, we could understand how to reconcile the quantum field theoretic description that is deemed necessary at the very early universe with the hydrodynamic/thermodynamic one, which suffices for the purposes of classical cosmology. It is the author's opinion that this perhaps is the only unambiguous way to establish the emergence of classicality in the cosmological context. It is also largely insensitive to the choice of interpretative scheme for quantum theory one chooses to use. At this stage, however, it is fair to say that there is no conclusive evidence about how the classical world appears and it is likely that a special initial condition is needed in order to guarantee it.

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